Sailboats, Ladders, and Old Glory

Commentary:

Materials required consist of a stick or rod (I like to use a meter stick), a weight, such as a hex nut and string to make a plumb bob, a spring scale or rubber band to measure forces up to a few times the weight of the stick (a 10-N scale works fine with a meter stick), and a protractor to measure angles.

In this exercise, we will explore the concepts of ***force*** (see "May the Force Be With You") as a **vector** quantity, ***center of gravity*** (see "Journey to the Center of Gravity"), and ***torque*** (see "Roll Out") in a very practical situation in which all three topics are synthesized. A force can be thought of as a push or pull and a torque as a twist about some pivot point. We will explore systems in ***rotational equilibrium***, that is, they are stable and not undergoing rotational acceleration about a pivot point. In simple static, or linear, equilibrium all of the forces are balanced so there is no net force. In the case of rotational equilibrium, all of the torques must balance or sum up to zero.

Have you ever had occasion to stand a ladder up against a wall? To raise a flagpole? To raise a mast on a sailboat? All of these situations and many others are ones in which we experience torque (the product of the applied perpendicular force and the moment arm, or distance between the force and pivot point) and actually feel physics concepts at work. In all three situations, you are probably standing on the ground and are limited as to how high on the object you can apply the force and the angle at which you can apply it (torque is maximized when the force is perpendicular to the moment arm and defined as F x R x sin of the angle between them). The sine function is there because only the component of a force vector perpendicular to the moment arm produces a torque. Remember that for very small angles, the sine function is zero and for 90 degrees it is one. It is never greater than one. To use a wrench as an example, it wouldn't work very well if you pushed straight down the wrench toward the nut you are trying to turn. It works best if you push at right angles to the wrench.

If you have done any of the above chores (raising ladders, poles, etc.) or anything similar, you found that it is easy to get started and the task becomes increasingly difficult as the ladder or pole is raised and suddenly becomes easy again when one reaches the vertical position. In your portfolio, comment on whether this agrees with your personal experience or if you have no such experiences with which to relate. We will investigate this process and show that these concepts are related to “body levers”, or the way our muscles make our bodies move.

Inquiry:

Let’s prepare by finding the center of gravity of the object to be raised. If it is straight and uniform like a meter stick, it should be located at the center. The center of gravity is the point at which gravity “acts on” the object. That is, from the point of view of gravity, all of the mass of the object can be considered as located at that point. Next, make a plumb bob with a short piece of string and a weight and attach the string to the center of gravity with a piece of tape, etc. The direction of the bob will always indicate the direction in which the force of gravity is acting on the object, producing a torque. You may wish to write the weight of the stick on a piece of paper and tape it to the suspended bob to help visualize the force vector which represents the force of gravity on the stick.

Part I

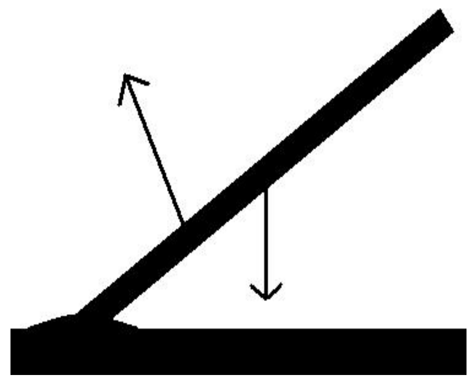
- Hold the stick so it is horizontal. In what direction does the bob point? What is the angle between the string and the stick?

- Hold the stick vertically and answer the same questions.

- Attach the spring scale to the center of the stick and record the weight of the stick, in Newtons. You may wish to write this value on the paper attached to the plumb bob as a reminder that gravity is always producing a torque that is “trying” to rotate the stick downward, toward the table when one end acts as a pivot point.

- Place one end of the stick on the table (this becomes a pivot point) and hold the stick at an angle somewhere in between and answer again. This time calculate the torque about the pivot end on the table due to the weight of the stick using torque = weight x sin of the angle between weight vector (bob) and the stick. Record your answer in Newton-meters.

- From now on, one partner will act as a pivot point at one end of the stick, so the other partner using the spring scale can raise the stick or rod. Let's make this as realistic as possible and attach the spring scale (or rubber band if you are measuring force by the amount of stretching) about one quarter to one third of the way up from the bottom of the stick.



- Begin raising the stick from horizontal to vertical and record the reading on the spring scale. Repeat this process for nearly horizontal and vertical and several angles in between. Act as though you we standing at the base and pushing on the ladder or pole simulated by the stick and apply the force in a direction that is also realistic. Record the angle between the scale and stick when you take each reading AND the angle between the plumb bob and the stick. You should end up with a table in your portfolio that looks similar to the one shown below but with more rows:

Angle of Stick (between stick and table)	**Angle Between Stick and Plumb Bob**	**Angle between Stick and Scale (applied force)**	**Applied Force (Newtons)**

- At what angle (height, or angle between stick and table) did it take the most force to hold it up?

- At what angle was the applied force a minimum?

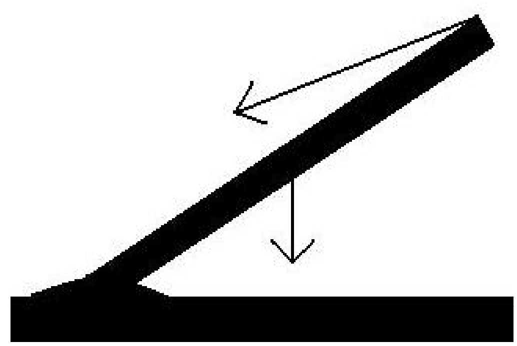
- Do your observations agree with practical experience or expectations for you?

- (Optional 1) Plot a graph of applied force as a function of the angular height of the stick. Is the graph linear or a curve? If it is a curve, at what angle do you see a maximum? Interpret the shape of the graph in terms of the physics involved in raising the object.

- (Optional 2) Calculate the torque due to the weight of the stick (F x R x sine of angle between stick and bob) and that due to the applied force for two angles. Are they equal in both cases? They should be if the stick is stationary or nearly so. This equality is called the ***second condition of equilibrium*** (if torques balance, there is no rotational acceleration).

Part II

- We will now repeat the steps followed in Part I except we will try an "engineering" experiment that you may have encountered in trying to raise a flagpole or mast or like the ancient Egyptians might have found while trying to figure out how to raise a obelisk. Bear in mind, once again, that the forces involved are small as we are performing a simulation with a lightweight stick but, in reality, they can be quite large! Let's test the "bright idea" that someone proposes to raise the object by tying a rope (string) to the top end and pulling on that to raise the object.



- This time your spring scale is connected to the top end of the stick, opposite the pivot end, with a piece of string. Again, pretend that you are standing on the ground, trying to raise the object. Measure the applied force for the nearly horizontal and vertical cases (uh, oh--can you even measure it for the horizontal case? Explain!) and for several angles in between and fill out the same type of table you did for part I:

Angular Height of Stick	Angle between Weight of Stick and Stick	Angle Between Stick and Applied Force	Applied Force (Newtons)

- This time, where was the applied force a maximum?

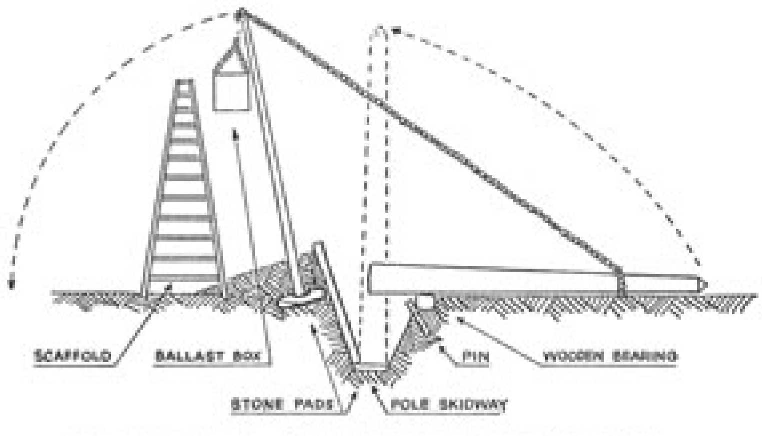
- At what angular height was the force a minimum?

- (Optional) Plot a graph of applied force as a function of the angular height of the object (as measured from the table). Did you get a straight line this time or a curve? Again, describe the shape of the graph in terms of the physics involved. Describe how this graph differs from that for Part I.

Part III

- **A contest!** You have been assigned the task of raising a long, heavy object (flag pole, etc.) and must use only manpower. “Play” with the positions of the applied force (you may use string as in Part II) to find the easiest way to raise the stick. The rules are simple. You may tie your “rope” for the applied force anywhere you wish. However, let’s assume that the object is at least four times as tall as you are high, so you cannot directly apply any force more than a quarter of the way up. After making your measurements, write down what you did to make the task easier and sketch your arrangement:

- Finally, we will try one other option for raising tall objects. You may have seen something like this in a drawing of ancient cultures erecting tall structures or by watching a crane at work. You are now allowed to have one more stick and incorporate it into your arrangement in any way you wish. In reality, many sailboats come with an extra pole to assist in raising the mast. Sketch your solution to ease the task and explain, **on the basis of torques** why it works.



COUNTERWEIGHTED A-FRAME FOR RAISING OBELISK

- Now for some human anatomy: The back muscles attach to the spine only a few centimeters away from the base of the spine. Thus, the angle between the back muscle and the spine is very small (much like the nearly horizontal stick in Part II). If I were to bend over so my torso is horizontal and try to lift a 50-pound weight by simply standing up, how small or large do you think the force exerted on the spine by the muscles must be? (You may do a rough calculation, or estimate, but should take into account everything you have observed about torques so far.) Can you understand why we are always advised to use our legs when lifting objects? Make a rough sketch to illustrate your answer, including all force vectors involved and explain your reasoning.